Assignment 3

2026

1. Obtain a three-term asymptotic approximation for the integral

$$I(\varepsilon) = \int_{\varepsilon}^{1} \sin(\varepsilon t)\mathrm{d}t,$$

as $\varepsilon \to 0$.

2. Obtain a three-term asymptotic approximation for the integral

$$J(\alpha) = \int_{0}^{\infty} \frac{5x - 3\alpha}{x + \alpha} e^{-x}\mathrm{d}x$$

where $\alpha \gg 1$ is a large parameter.

3. Use Laplace's method to obtain a leading order approximation of the integral

$$\int_{0}^{2\pi} (t^2 + 1)e^{-\lambda(2+\sin t)}\mathrm{d}t$$

as $\lambda \to \infty$.

4. Find a two-term asymptotic approximation of a periodic solution of the Duffing equation

$$\ddot{x} + 4x - 2\varepsilon x^3 = 0,$$

with initial conditions

$$x(0) = A, \qquad \dot{x}(0) = 0,$$

where ε is a small parameter.